1 Solve the simultaneous equations

$$7x + 2y = 5.5$$
$$3x - 5y = 17$$

Show clear algebraic working.

$$7x + 2y = 5.5$$
$-2y$
$$7x = 5.5 - 2y$$
$\div 7$
$$x = \frac{5.5 - 2y}{7} \quad 1 \quad — 1$$

substitute 1 into $3x - 5y = 17$

$$3\left(\frac{5.5 - 2y}{7}\right) - 5y = 17 \quad 1$$
$\times 7$
$$16.5 - 6y - 35y = 119$$
-16.5
$$-6y - 35y = 119 - 16.5$$
$$-41y = 102.5$$
$\div -41$
$$y = -2.5 \quad 1$$

substitute $y = -2.5$ into 1

$$x = \frac{5.5 - 2(-2.5)}{7}$$
$$= 1.5 \quad 1$$

$x =$ 1.5

$y =$ -2.5

(Total for Question 1 is 4 marks)

2 Solve the simultaneous equations

$$7x - 2y = 34$$
$$3x + 5y = -3$$

Show clear algebraic working.

$3x + 5y = -3$ — ②

$7x - 2y = 34$

$2y = 7x - 34$

$y = \frac{7x - 34}{2}$ — ①

substitute ① into ②

$3x + 5\left(\frac{7x - 34}{2}\right) = -3$ ①

$6x + 35x - 170 = -6$

$41x = -6 + 170$ ①

$x = \frac{164}{41}$

$= 4$

$y = \frac{7(4) - 34}{2}$ ①

$= \frac{28 - 34}{2}$

$= -3$

$x =$ 4 ①

$y =$ -3

(Total for Question 2 is 4 marks)

3 Solve the simultaneous equations

$$3xy - y^2 = 8 \quad — 1$$
$$x - 2y = 1$$

Show clear algebraic working.

$x = 1 + 2y \quad — 2$

Substitute 2 into 1 :

$3(1+2y)y - y^2 = 8$ 1

$3y + 6y^2 - y^2 = 8$

$5y^2 + 3y - 8 = 0$ 1

$$y = \frac{-3 \pm \sqrt{3^2 - 4(5)(-8)}}{2(5)}$$ 1

$$= \frac{-3 \pm \sqrt{169}}{10}$$

$$= \frac{-3 \pm 13}{10}$$

$y = 1$ or $y = -\frac{8}{5}$ – substitute into 2

$x = 1 + 2(1)$ or $x = 1 + 2\left(-\frac{8}{5}\right)$ 1

$= 3$ $\qquad = -\frac{11}{5}$

1

$x = 3,\ y = 1$ and $x = \frac{-11}{5},\ y = \frac{-8}{5}$

(Total for Question 3 is 5 marks)

4 Solve the simultaneous equations

$$3x + 5y = 6$$
$$7x - 5y = -11 \quad \text{—} ①$$

$x = \dfrac{6-5y}{3}$ — ②

Show clear algebraic working.

Substitute ② into ① :

$7\left(\dfrac{6-5y}{3}\right) - 5y = -11$

$7(6-5y) - 15y = -33$

$42 - 35y - 15y = -33$

$-50y = -33 - 42$ $\quad -42$

$-50y = -75$ ①

$\div 50$

$y = \dfrac{-75}{-50} = 1.5$ ①

$x = \dfrac{6-5(1.5)}{3}$

$= -0.5$ ①

$x =$ .... -0.5 ....

$y =$ .... 1.5 ....

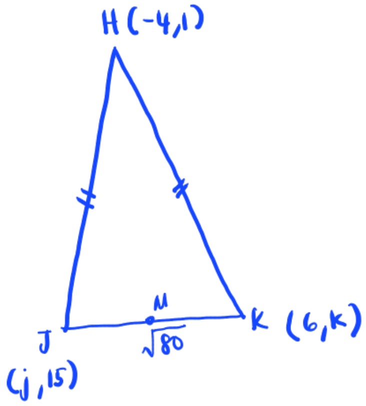
(Total for Question 4 is 3 marks)

5 Triangle HJK is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

H is the point with coordinates $(-4, 1)$
J is the point with coordinates $(j, 15)$ where $j < 0$
K is the point with coordinates $(6, k)$

M is the midpoint of JK.
The gradient of HM is 2

Find the value of j and the value of k.



Given : gradient of HM = 2

gradient of JK $= \frac{-1}{2} = -\frac{1}{2}$ (1)

$-\frac{1}{2} = \frac{(k-15)}{(6-j)}$

$-6 + j = 2k - 30$

$j = 2k - 24$ — (1) (1)

Given : length of JK $= \sqrt{80}$

$\sqrt{(6-j)^2 + (k-15)^2} = \sqrt{80}$

$(6-j)^2 + (k-15)^2 = 80$ (1)

$j^2 - 12j + 36 + k^2 - 30k + 225 = 80$

$j^2 - 12j + k^2 - 30k = -181$ — (2)

substitute (1) into (2) :

$(2k-24)^2 - 12(2k-24) + k^2 - 30k = -181$

$4k^2 - 96k + 576 - 24k + 288 + k^2 - 30k = -181$

$5k^2 - 150k + 1045 = 0$ (1)

$k = \frac{150 \pm \sqrt{(-150)^2 - 4(5)(1045)}}{2(5)}$ (1)

$= \frac{150 \pm \sqrt{1600}}{10}$

$= \frac{150 \pm 40}{10}$

$k = 19$ or 11

substitute k values into ①

$j = 2(19) - 24$ or $j = 2(11) - 24$

$= 14$ or $j = -2$

since $j < 0$,

$\therefore$ $j = -2$ and $k = 11$ ①

$j =$ -2

$k =$ 11

(Total for Question 5 is 6 marks)

6 The line with equation $y = x + 2$ intersects the curve with equation $x^2 + y^2 - 2y = 24$ at the points A and B.

Find the coordinates of A and B.
Show clear algebraic working.

By using simultaneous equation:

$x^2 + (x+2)^2 - 2(x+2) = 24$ 1

$x^2 + x^2 + 4x + 4 - 2x - 4 = 24$

$2x^2 + 2x = 24$

$x^2 + x - 12 = 0$ 1

$(x-3)(x+4) = 0$ 1

$x = 3$ or $x = -4$ 1 — Substitute into $y = x+2$ to get y values.

$y = 5$ or $y = -2$

(3 , 5) 1

(−4 , −2)

(Total for Question 6 is 5 marks)

7 Given that $x = \dfrac{5}{9y+5}$ and that $y = \dfrac{5}{5a-2}$

find an expression for x in terms of a.
Give your expression as a single fraction in its simplest form.

$$x = \frac{5}{9\left(\frac{5}{5a-2}\right)+5} \quad (1)$$

$$= \frac{5}{\frac{45}{5a-2} + \frac{5(5a-2)}{5a-2}}$$

$$= \frac{5}{\frac{45+25a-10}{5a-2}} \quad (1)$$

$$= \frac{5(5a-2)}{35+25a}$$

$$= \frac{5(5a-2)}{5(7+5a)} \quad (1)$$

$$x = \frac{5a-2}{7+5a} \quad (1)$$

$x = \dfrac{5a-2}{7+5a}$

(Total for Question 7 is 4 marks)

8 Solve the simultaneous equations

$$5a + 2c = 10 \quad - 1$$
$$2a - 4c = 7$$

$\div 2$

$$a - 2c = \frac{7}{2} \quad - 2$$

Show clear algebraic working.

substitute 2 into 1 :

$$5\left(\frac{7}{2} + 2c\right) + 2c = 10 \quad 1$$

$$\frac{35}{2} + 10c + 2c = 10$$

$$\frac{35}{2} + 12c = 10$$

$$12c = 10 - \frac{35}{2} \quad 1$$

$$c = \frac{-7.5}{12}$$

$$= -0.625$$

$$a = \frac{7}{2} + 2(-0.625)$$

$$= 2.25 \quad 1$$

$a =$ 2.25

$c =$ -0.625

(Total for Question 8 is 3 marks)

9 Solve the simultaneous equations

$$y = 3 - 2x \quad - 1$$
$$x^2 + y^2 = 18 \quad - 2$$

Show clear algebraic working.

substitute 1 into 2 :

$x^2 + (3-2x)^2 = 18$ (1)

$x^2 + 9 - 12x + 4x^2 = 18$

$5x^2 - 12x + 9 - 18 = 0$

$5x^2 - 12x - 9 = 0$ (1)

$x = \dfrac{12 \pm \sqrt{(-12)^2 - 4(5)(-9)}}{2(5)}$ (1)

$= \dfrac{12 \pm \sqrt{324}}{10}$

$= \dfrac{12 \pm 18}{10}$

$x = 3$ or $x = -0.6$

$y = -3$ or $y = 4.2$ (1)

$x = 3,\ y - 3$, $x = -0.6,\ y = 4.2$ (1)

(Total for Question 9 is 5 marks)

10 Solve the simultaneous equations

$$x - 6y = 5 \quad x = 5 + 6y \; -(1)$$
$$xy - 2y^2 = 6 \; -(2)$$

Show clear algebraic working.

Substitute (1) into (2) :

$(5+6y)y - 2y^2 = 6$ (1)

$5y + 6y^2 - 2y^2 = 6$

$4y^2 + 5y - 6 = 0$ (1)

$(4y-3)(y+2) = 0$ (1)

$y = \frac{3}{4}$ or $y = -2$ (1)

Substitute y values into (1) :

$x = 5 + 6\left(\frac{3}{4}\right)$ or $x = 5 + 6(-2)$

$x = \frac{19}{2}$ or $x = -7$

(1)

$x = \frac{19}{2}$, $y = \frac{3}{4}$, $x = -7$, $y = 2$

(Total for Question 10 is 5 marks)

11 Solve the simultaneous equations $2x + 7y = 17$
$5x + 3y = -1$

Show clear algebraic working.

$2x + 7y = 17$

$2x = 17 - 7y$

$x = \frac{17 - 7y}{2}$ — 1

$5x + 3y = -1$

$5x = -1 - 3y$

$x = \frac{-1 - 3y}{5}$ — 2

Substitute 2 into 1

$\frac{-1 - 3y}{5} = \frac{17 - 7y}{2}$

$2(-1 - 3y) = 5(17 - 7y)$ 1

$-2 - 6y = 85 - 35y$

$-2 - 85 = -35y + 6y$

$-87 = -29y$

$y = 3$

$x = \frac{-1 - 3(3)}{5} = -2$ 1

$x =$ .... -2 1

$y =$ 3 1

(Total for Question 11 is 4 marks)

12 Solve the simultaneous equations

$$x^2 - 9y - x = 2y^2 - 12 \quad \text{— } ②$$
$$x + 2y - 1 = 0$$

Show clear algebraic working.

$x + 2y - 1 = 0$

$x = 1 - 2y$ — ①

Substitute ① into ②

$(1-2y)^2 - 9y - (1-2y) = 2y^2 - 12$ (1)

$1 - 4y + 4y^2 - 9y - 1 + 2y = 2y^2 - 12$

$4y^2 - 11y = 2y^2 - 12$

$2y^2 - 11y + 12 = 0$ (1)

$(2y-3)(y-4) = 0$ (1)

$y = \frac{3}{2}$ or $y = 4$ (1)

$x = 1 - 2\left(\frac{3}{2}\right)$ or $x = 1 - 2(4)$

$x = -2$ or $x = -7$

$x = -2,\ y = \frac{3}{2}$, $x = -7,\ y = 4$ (1)

(Total for Question 12 is 5 marks)

13 Solve the simultaneous equations

$$3x - 5y = 25 \quad - (1)$$
$$4x + 3y = 14$$

Show clear algebraic working.

$$x = \frac{14 - 3y}{4} \quad - (2)$$

Substitute (2) into (1) :

$$3\left(\frac{14-3y}{4}\right) - 5y = 25 \quad (1)$$

$$3(14-3y) - 5y(4) = 25(4)$$

$$42 - 9y - 20y = 100$$

$$-29y = 100 - 42$$

$$-29y = 58$$

$$y = \frac{58}{-29} = -2 \quad (1)$$

Substitute $y = -2$ into (2)

$$x = \frac{14 - 3(-2)}{4} \quad (1)$$

$$= 5$$

$x =$ 5 (1)

$y =$ -2

(Total for Question 13 is 4 marks)

14 The sum of the first 10 terms of an arithmetic series is 4 times the sum of the first 5 terms of the same series.

The 8th term of this series is 45

Find the first term of this series.
Show clear algebraic working.

$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$S_{10} = \frac{10}{2}\left[2a+(10-1)d\right]$

$= 10a + 45d$

$S_5 = \frac{5}{2}\left[2a+(5-1)d\right]$

$= 5a + 10d$

$S_{10} = 4 \times S_5$

$10a + 45d = 4(5a+10d)$ (1)

$10a + 45d = 20a + 40d$

$10a = 5d$

$d = 2a$ — (1) (1)

$T_n = a + (n-1)d$

$T_8 = 45 = a + (8-1)d$

$45 = a + 7d$ — (2) (1)

substitute (1) into (2) :

$45 = a + 7(2a)$ (1)

$45 = 15a$

$a = 3$ (1)

$\therefore$ First term, $a = 3$.

3

(Total for Question 14 is 5 marks)

15 Solve the simultaneous equations

$$x - 2y = 3$$
$$x^2 - y^2 + 2x = 10 \quad - ①$$

Show clear algebraic working.

$$x = 2y + 3 \quad - ②$$

substitute ② into ① :

$(2y+3)^2 - y^2 + 2(2y+3) = 10$ ①

$4y^2 + 12y + 9 - y^2 + 4y + 6 = 10$

$3y^2 + 16y + 5 = 0$ ①

$(3y+1)(y+5) = 0$ ①

$y = -\frac{1}{3}$, $y = -5$

substitute y values into ②

$x = 2(-\frac{1}{3}) + 3$, $x = 2(-5) + 3$ ①

$x = \frac{7}{3}$, $x = -7$

①

$x = \frac{7}{3}, y = -\frac{1}{3}$ and $x = -7, y = -5$

(Total for Question 15 is 5 marks)

16 Solve the simultaneous equations

$$7x + 3y = 3 \xrightarrow{\times 3} 21x + 9y = 9$$
$$3x - y = 7 \xrightarrow{\times 7} 21x - 7y = 49$$

Show clear algebraic working.

$9y - (-7y) = 9 - 49$ (1)

$16y = -40$

$y = \frac{-40}{16}$

$= -2.5$

$3x + 2.5 = 7$ (1)

$3x = 4.5$

$x = \frac{4.5}{3} = 1.5$

$x =$ 1.5 (1)

$y =$ -2.5

(Total for Question 16 is 3 marks)

17 The line with equation $2y = x + 1$ intersects the curve with equation $3y^2 + 7y + 16 = x^2 - x$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

$3y^2 + 7y + 16 = (2y-1)^2 - (2y-1)$ 1

$3y^2 + 7y + 16 = 4y^2 - 4y + 1 - 2y + 1$

$3y^2 - 4y^2 + 7y + 6y + 16 - 2 = 0$

$-y^2 + 13y + 14 = 0$

$y^2 - 13y - 14 = 0$ 1

$(y-14)(y+1) = 0$ 1

$y = 14$, $y = -1$

$x = 2(14) - 1$, $x = 2(-1) - 1$
$= 27$ $= -3$ 1

$(27, 14)$ and $(-3, -1)$
1

(27 , 14) and (−3 , −1)

(Total for Question 17 is 5 marks)

18 Solve the simultaneous equations

$$3x^2 + y^2 - xy = 5$$
$$y = 2x - 3$$

Show clear algebraic working.

$3x^2 + (2x-3)^2 - x(2x-3) = 5$ (1)

$3x^2 + 4x^2 - 12x + 9 - 2x^2 + 3x - 5 = 0$

$5x^2 - 9x + 4 = 0$ (1)

$(5x-4)(x-1) = 0$

$x = \frac{4}{5}$, $x = 1$ (1)

$y = 2\left(\frac{4}{5}\right) - 3$, $y = 2(1) - 3$

$= -\frac{7}{5}$, -1 (1)

(1)

$x = 0.8, y = -1.4$ / $x = 1, y = -1$

(Total for Question 18 is 5 marks)

19 Solve the simultaneous equations

$$\times 2$$

$3x + 5y = 3.1$ $6x + 10y = 6.2$ – 1

$6x + 3y = 3.75$ – 2

Show clear algebraic working.

By elimination:

1 – 2 :

$10y - 3y = 6.2 - 3.75$

$7y = 2.45$ 1

$y = 0.35$

$3x + 5(0.35) = 3.1$ 1

$3x + 1.75 = 3.1$

$3x = 1.35$

$x = 0.45$

$x =$ 0.45 1

$y =$ 0.35

(Total for Question 19 is 3 marks)

20 An arithmetic series has first term a and common difference d, where d is a prime number.

The sum of the first n terms of the series is S_n and

$S_m = 39$

$S_{2m} = 320$

Find the value of d and the value of m
Show clear algebraic working.

$S_m = \frac{m}{2}\left[2a + (m-1)d\right] = 39$ (1)

$2am + m^2d - md = 78$ — (1)

$S_{2m} = \frac{\cancel{2}m}{\cancel{2}}\left[2a + (2m-1)d\right] = 320$ (1)

$= 2am + 2m^2d - md = 320$ — (2)

(2) − (1) :

$m^2d = 320 - 78$

$m^2d = 242$ (1)

if $d = 2$: $m^2 = \frac{242}{2}$ (1)

$m^2 = 121$

$m = 11$

$d =$ 2 (1)

$m =$ 11

(Total for Question 20 is 5 marks)

21 Solve the simultaneous equations

$$x + 2y = 15 \qquad x = 15 - 2y \quad -(1)$$
$$4x - 6y = 4 \quad -(2)$$

Show clear algebraic working.

subs (1) into (2):

$4(15 - 2y) - 6y = 4$

$60 - 8y - 6y = 4$ (1)

$56 = 14y$

$y = 4$

$x = 15 - 2(4)$ (1)

$= 7$

$x =$ 7 (1)

$y =$ 4

(Total for Question 21 is 3 marks)

22 Solve the simultaneous equations

$$2y^2 + x^2 = -6x + 42 \quad — ①$$
$$2x + y = -3$$

$$y = -3 - 2x \quad — ②$$

Show clear algebraic working.

$2(-3-2x)^2 + x^2 = -6x + 42$ (1)

$2(9 + 12x + 4x^2) + x^2 = -6x + 42$

$18 + 24x + 8x^2 + x^2 = -6x + 42$

$9x^2 + 30x - 24 = 0$ $\div 3$

$3x^2 + 10x - 8 = 0$ (1)

$(3x-2)(x+4) = 0$ (1)

$x = \frac{2}{3}, -4$ (1)

$y = -3 - 2\left(\frac{2}{3}\right)$, $y = -3 - 2(-4)$

$y = \frac{-13}{3}$, $y = 5$

(1)

$x = \frac{2}{3}, y = \frac{-13}{3}$ and $x = -4, y = 5$

(Total for Question 22 is 5 marks)

23 Solve the simultaneous equations

$$5x + 4y = -2 \quad (1)$$
$$2x - y = 4.4$$

Show clear algebraic working.

$y = 2x - 4.4 \quad (2)$

$5x + 4(2x - 4.4) = -2 \quad (1)$

$5x + 8x - 17.6 = -2$

$13x = 15.6 \quad (1)$

$x = \frac{15.6}{13}$

$= 1.2$

$y = 2(1.2) - 4.4$

$= 2.4 - 4.4 \quad (1)$

$= -2$

$x =$ 1.2

$y =$ -2

(Total for Question 23 is 3 marks)

24 Solve the simultaneous equations

$$y = 7 - 2x \quad - ①$$
$$x^2 + y^2 = 34 \quad - ②$$

Show clear algebraic working.

① into ② :

$x^2 + (7-2x)^2 = 34$ (1)

$x^2 + 49 - 28x + 4x^2 = 34$

$5x^2 - 28x + 15 = 0$ (1)

$(5x-3)(x-5) = 0$ (1)

$x = 0.6 , x = 5$

$y = 7 - 2(0.6)$, $y = 7 - 2(5)$ (1)

$= 5.8$ $= -3$

(1)

$x = 0.6, y = 5.8$, $x = 5, y = -3$

(Total for Question 24 is 5 marks)

25 Work out the coordinates of the points of intersection of

$$y - 2x = 1 \quad \text{and} \quad y^2 + xy = 7 \quad - \text{②}$$

Show clear algebraic working. $y = 2x + 1 \quad - \text{①}$

substitute ① into ②

$(2x+1)^2 + (2x+1)x = 7$ ①

$4x^2 + 4x + 1 + 2x^2 + x = 7$

$6x^2 + 5x - 6 = 0$ ①

$(2x+3)(3x-2) = 0$ ①

$x = -\frac{3}{2}$ and $x = \frac{2}{3}$

substitute x values into ① :

$y = 2\left(-\frac{3}{2}\right) + 1$ and $y = 2\left(\frac{2}{3}\right) + 1$ ①

$= -2$ and $\frac{7}{3}$

①

($-\frac{3}{2}$, -2)

($\frac{2}{3}$, $\frac{7}{3}$)

(Total for Question 25 is 5 marks)

26 The straight line with equation $y - 2x = 7$ is the perpendicular bisector of the line AB where A is the point with coordinates $(j, 7)$ and B is the point with coordinates $(6, k)$

Find the coordinates of the midpoint of the line AB
Show clear algebraic working.

$y = 2x + 7$

$m = 2$

$m_{AB} = -\frac{1}{2}$ (1)

$-\frac{1}{2} = \frac{k-7}{6-j}$ (1)

$-6 + j = 2k - 14$

$2k - j = 8$ — (1)

midpoint of AB : $\left(\frac{j+6}{2}, \frac{7+k}{2}\right)$ (1)

$\frac{7+k}{2} = \cancel{2}\left(\frac{j+6}{\cancel{2}}\right) + 7$

(1)

$7 + k = 2j + 12 + 14$

$k = 2j + 19$ — (2)

substitute (2) into (1) :

$2(2j + 19) - j = 8$

$4j + 38 - j = 8$

$3j = -30$

$j = -10$ (1)

$k = 2(-10) + 19$

$= -1$

midpoint of AB : $\left(\frac{-10+6}{2}, \frac{7-1}{2}\right)$

$= (-2, 3)$ (1)

(-2 , 3)

(Total for Question 26 is 6 marks)

27 Solve the simultaneous equations

$$2x + 9y = 14.5 \quad — \ 1$$
$$7x + 3y = 8$$

$\times 3$ $\quad 21x + 9y = 24$ $\quad \times 3 \quad — \ 2$

Show clear algebraic working.

2 − 1 :

$21x - 2x + 9y - 9y = 24 - 14.5$

1

$19x = 9.5$

$x = \frac{9.5}{19} = \frac{1}{2}$

$2(\frac{1}{2}) + 9y = 14.5$

1

$1 + 9y = 14.5$

$9y = 13.5$

$y = \frac{13.5}{9} = 1.5$

$x =$ 0.5 1

$y =$ 1.5

(Total for Question 27 is 3 marks)

28 Solve the simultaneous equations

$$2x^2 + 3y^2 = 11$$
$$x = 3y - 1$$

Show clear algebraic working.

$2(3y-1)^2 + 3y^2 = 11$ (1)

$2(9y^2 - 6y + 1) + 3y^2 = 11$

$18y^2 - 12y + 2 + 3y^2 = 11$

$21y^2 - 12y - 9 = 0$ (1)

$7y^2 - 4y - 3 = 0$

$(7y + 3)(y - 1) = 0$ (1)

$y = -\frac{3}{7}$ and $y = 1$

$x = 3\left(-\frac{3}{7}\right) - 1$ and $x = 3(1) - 1$

$x = \frac{-16}{7}$ and $x = 2$ (1)

(1)

$x = 2, y = 1$ and $x = \frac{-16}{7}, y = -\frac{3}{7}$

(Total for Question 28 is 5 marks)